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# ANALYSIS OF THE BOUNDARY CONDITIONS OF THE COMPLEX MODEL OF MASS EXCHANGE IN RECTIFICATION PROCESSES

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*Boundary conditions of applicability of the complex mass-exchange model to forward, backward, and cross motions of the vapor and liquid phases in rectification have been found. Four variants of mass exchange which are the limits of the complex model have been considered. The limiting values of the distances h and $h_1$ from the site of injection of the phases at which the compositions in ideal and real plates are equalized have been computed. The relations between the basic technological parameters of the rectification process under the boundary conditions have been obtained.*

The complex model [1–3] differs from the well-known models of Murphree and Hausen [4–6] in that the compositions of the flows on ideal and real plates are equalized at a certain distance $h$ (for a vapor) and $h_1$ (for a liquid) from the site of injection of the phases. In the Murphree model, in analyzing the efficiency in the vapor phase and the liquid these distances take on the values

$$h = 0\,,\quad h_1 = 1\,; \tag{1}$$

$$h = 1\,,\quad h_1 = 0\,; \tag{2}$$

in the Hausen model, they are

$$h = 0\,,\quad h_1 = 0 \tag{3}$$

and in the hypothetical model obtained from analysis of the possible variants of interrelationship between the ideal and real plates, these distances are equal to

$$h = 1\,,\quad h_1 = 1\,. \tag{4}$$

Furthermore, the complex model provides for the variant of separation of an ideal mixture in which the distances $h$ and $h_1$ are equal to

$$h = h_1 = 0.5\,. \tag{5}$$

In [7], as a result of the analysis of the interrelationship between individual parameters we have proposed the relation

$$h = h_1 = \frac{1}{m+1}\,. \tag{6}$$

Thus, in the complex model the distances $h$ and $h_1$ depend on the coefficient of phase equilibrium (5). For nearly ideal mixtures in which $m$ tends to unity, $h$ and $h_1$ are determined from formula (5). In separation of mixtures with an increasing coefficient of phase equilibrium, the distances $h$ and $h_1$ decrease and become equal to zero in the

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limit where $m$ tends to infinity, which is reflected by formula (3). In separation of a mixture with a decreasing coefficient of phase equilibrium, the distances $h$ and $h_1$ increase and become equal to unity when $m = 0$, as is observed in the hypothetical model to which equalities (4) belong. In the case of separation of mixtures in which the coefficient of phase equilibrium simultaneously increases as applied to the vapor phase and decreases when mass exchange in the liquid is analyzed, the distance $h$ decreases and the distance $h_1$ increases; finally, they take on values in accordance with (1), which is expressed by the Murphree model in analyzing the efficiency in the vapor phase. In the case of the reverse trend whose limit is formula (2), the Murphree model is obtained in analyzing the efficiency in the liquid.

The decrease in $h$ and the increase in $h_1$ with increase in the coefficient of phase equilibrium or their opposite changes with decrease in $m$ point to the discrepancy between these regions of the complex model and the corresponding boundary cases, i.e., the Murphree model, in analyzing the efficiency in the vapor phase and the liquid, since mass exchange occurs in one mixture and the intensification of the release of a highly volatile component from the liquid is equivalent to the rate of enrichment of the vapor phase with this component; this must result in a simultaneous decrease in $h$ and $h_1$. However, the efficiency of mass exchange in the Murphree model has received the widest acceptance, in direct form or indirectly, in analyzing mass-exchange processes despite its drawbacks noted by different researchers. Therefore, both variants of the Murphree model are involved in analysis interchangeably with others.

The Murphree and Hausen models are applicable in complete mixing of the liquid on a plate or in forward (concurrent) motion of the flows. In [2, 3], the conditions of interrelationship between the ideal and real plates which correspond to the Murphree and Hausen model have been extended to the backward (countercurrent) and cross motions of the vapor and the liquid. The corresponding conditions of interrelationship between the ideal and real plates are specified by variants. The first two variants are characterized by the conditions of the Murphree model (1) and (2), the third variant is characterized by those of the Hausen model (3), and the fourth variant is characterized by the conditions of the hypothetical model (4).

The absence of the real values of the efficiency in certain variants of mass exchange has been revealed in [1–3]. In this connection, it is expedient to find the validity range for the entire complex model and its individual, boundary variants, in particular, in forward flow, backward flow, and cross flow of the vapor and the liquid.

For the forward motion of the phases, from formula (14) [1] we have derived the dependence

$$\frac{x_{n-1} - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}} E_f = \frac{L}{mV} + (1 - E_f)\left(1 - h\frac{L}{mV} - h_1\right). \tag{7}$$

By subtracting $E_f L/(mV)$ from the left-hand and right-hand sides (7) and with account for the material-balance equation $L(x_n - x_{n-1}) = V(y_n - y_{n-1})$, we have obtained the relation

$$\frac{x_{n-1} - \dfrac{y_n}{m}}{x_n - x_{n-1}} E_f = (1 - E_f)\left(\frac{L}{mV} + 1 - h\frac{L}{mV} - h_1\right). \tag{8}$$

After the addition of $E_f$ to both sides of formula (7), we have found the expression

$$\frac{x_n - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}} E_f = \frac{L}{mV} + 1 - (1 - E_f)\left(h\frac{L}{mV} + h_1\right). \tag{9}$$

Subtraction of $E_f[L/(mV) - 1]$ from both sides of (7) leads to the formula

$$\frac{x_n - \dfrac{y_n}{m}}{x_n - x_{n-1}} E_f = 1 + (1 - E_f)\left(\frac{L}{mV} - h\frac{L}{mV} - h_1\right). \tag{10}$$

For the backward motion of the phases, from (3) [2] we have derived the relation

$$\frac{x_{n-1}-\dfrac{y_{n-1}}{m}}{x_n-x_{n-1}}E_{\mathrm{g}}=\frac{L}{mV}-E_{\mathrm{g}}-(1-E_{\mathrm{g}})\left(h\,\frac{L}{mV}+h_1\right). \tag{11}$$

In just the same manner as in the case of forward flow, we have derived from (11) equations which are analogous to (8)–(10) in backward flow:

$$\frac{x_{n-1}-\dfrac{y_n}{m}}{x_n-x_{n-1}}E_{\mathrm{g}}=(1-E_{\mathrm{g}})\left(\frac{L}{mV}+1-h\,\frac{L}{mV}-h_1\right)-1\,, \tag{12}$$

$$\frac{x_n-\dfrac{y_{n-1}}{m}}{x_n-x_{n-1}}E_{\mathrm{g}}=\frac{L}{mV}-(1-E_{\mathrm{g}})\left(h\,\frac{L}{mV}+h_1\right), \tag{13}$$

$$\frac{x_n-\dfrac{y_n}{m}}{x_n-x_{n-1}}E_{\mathrm{g}}=(1-E_{\mathrm{g}})\left(\frac{L}{mV}-h\,\frac{L}{mV}-h_1\right). \tag{14}$$

In cross motion of the phases, by analogy with (7)–(10) we have derived from formula (7) [3] the dependences

$$\frac{x_{n-1}-\dfrac{y_{n-1}}{m}}{x_n-x_{n-1}}E_{\mathrm{k}}=\frac{L}{mV}-\frac{1}{2}+(1-E_{\mathrm{k}})\left(1-h\,\frac{L}{mV}-h_1\right), \tag{15}$$

$$\frac{x_{n-1}-\dfrac{y_n}{m}}{x_n-x_{n-1}}E_{\mathrm{k}}=(1-E_{\mathrm{k}})\left(\frac{L}{mV}+1-h\,\frac{L}{mV}-h_1\right)-\frac{1}{2}\,, \tag{16}$$

$$\frac{x_n-\dfrac{y_{n-1}}{m}}{x_n-x_{n-1}}E_{\mathrm{k}}=\frac{L}{mV}+\frac{1}{2}-(1-E_{\mathrm{k}})\left(h\,\frac{L}{mV}+h_1\right), \tag{17}$$

$$\frac{x_n-\dfrac{y_n}{m}}{x_n-x_{n-1}}E_{\mathrm{k}}=(1-E_{\mathrm{k}})\left(\frac{L}{mV}-h\,\frac{L}{mV}-h_1\right)+\frac{1}{2}\,. \tag{18}$$

Upon substitution of the values of the distances $h$ and $h_1$ from (1)–(5) into Eqs. (7)–(18), we have derived the ratios of the concentration difference for all the variants of mass exchange in forward flow, backward flow, and cross flow of the interacting phases (Table 1).

Table 1 shows that in forward flow, the considered ratios of the concentration difference in all the variants have positive values. In the second variant, ratio b, which can be negative for $E_{\mathrm{f}} > 0.5$, is somewhat doubted in this respect; this points to certain limitations.

In the fourth variant, the ratio of the concentration difference is independent of the efficiency. The equality of ratio b to zero and of ratio d to unity in combination with the conditions of equilibrium of the flows after the ideal plate $y_n^* = mx_{n-1}^*$ points to the fact that, in the fourth variant, the real plate is analogous to the ideal plate. Conse-

TABLE 1. Boundary Conditions of the Complex Model for Different Forms of Organization of Flows

| Ratio | Variants of mass exchange | | | | |
|---|---|---|---|---|---|
| | 1 ($h = 0$; $h_1 = 1$) | 2 ($h = 1$; $h_1 = 0$) | 3 ($h = 0$; $h_1 = 0$) | 4 ($h = 1$; $h_1 = 1$) | ideal mixture ($h = h_1 = 0.5$) |
| *Forward flow* | | | | | |
| a) $\dfrac{x_{n-1}-\frac{y_{n-1}}{m}}{x_n-x_{n-1}}$ | $\frac{L}{mV}\frac{1}{E_{\mathrm{f1}}}$ | $\frac{L}{mV}+\frac{1}{E_{\mathrm{f2}}}-1$ | $\left(\frac{L}{mV}+1\right)\frac{1}{E_{\mathrm{f3}}}-1$ | $\frac{L}{mV}$ | $\frac{1}{2}\left[\frac{L}{mV}\left(\frac{1}{E_{\mathrm{f,m}}}-1\right)+\frac{1}{E_{\mathrm{f,m}}}-1\right]$ |
| b) $\dfrac{x_{n-1}-\frac{y_n}{m}}{x_n-x_{n-1}}$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{f1}}}-1\right)$ | $\frac{1}{E_{\mathrm{f2}}}-1$ | $\left(\frac{L}{mV}+1\right)\left(\frac{1}{E_{\mathrm{f3}}}-1\right)$ | 0 | $\frac{1}{2}\left(\frac{L}{mV}+1\right)\left(\frac{1}{E_{\mathrm{f,m}}}-1\right)$ |
| c) $\dfrac{x_n-\frac{y_{n-1}}{m}}{x_n-x_{n-1}}$ | $\frac{L}{mV}\frac{1}{E_{\mathrm{f1}}}+1$ | $\frac{L}{mV}+\frac{1}{E_{\mathrm{f2}}}$ | $\left(\frac{L}{mV}+1\right)\frac{1}{E_{\mathrm{f3}}}$ | $\frac{L}{mV}+1$ | $\frac{1}{2}\left(\frac{L}{mV}+1\right)\left(\frac{1}{E_{\mathrm{f,m}}}+1\right)$ |
| d) $\dfrac{x_n-\frac{y_n}{m}}{x_n-x_{n-1}}$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{f1}}}-1\right)+1$ | $\frac{1}{E_{\mathrm{f2}}}$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{f3}}}-1\right)+\frac{1}{E_{\mathrm{f3}}}$ | 1 | $\frac{1}{2}\left[\frac{L}{mV}\left(\frac{1}{E_{\mathrm{f,m}}}-1\right)+\frac{1}{E_{\mathrm{f,m}}}+1\right]$ |
| *Backward flow* | | | | | |
| a) $\dfrac{x_{n-1}-\frac{y_{n-1}}{m}}{x_n-x_{n-1}}$ | $\left(\frac{L}{mV}-1\right)\frac{1}{E_{\mathrm{g1}}}$ | $\frac{L}{mV}-1$ | $\frac{L}{mV}\frac{1}{E_{\mathrm{g3}}}-1$ | $\frac{L}{mV}-\frac{1}{E_{\mathrm{g4}}}$ | $\frac{1}{2}\left(\frac{L}{mV}-1\right)\left(\frac{1}{E_{\mathrm{g,m}}}+1\right)$ |
| b) $\dfrac{x_{n-1}-\frac{y_n}{m}}{x_n-x_{n-1}}$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{g1}}}-1\right)-\frac{1}{E_{\mathrm{g1}}}$ | –1 | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{g3}}}-1\right)-1$ | $-\frac{1}{E_{\mathrm{g4}}}$ | $\frac{1}{2}\left[\left(\frac{L}{mV}-1\right)\frac{1}{E_{\mathrm{g,m}}}-\frac{L}{mV}-1\right]$ |
| c) $\dfrac{x_n-\frac{y_{n-1}}{m}}{x_n-x_{n-1}}$ | $\left(\frac{L}{mV}-1\right)\frac{1}{E_{\mathrm{g1}}}+1$ | $\frac{L}{mV}$ | $\frac{L}{mV}\frac{1}{E_{\mathrm{g3}}}$ | $\frac{L}{mV}-\frac{1}{E_{\mathrm{g4}}}+1$ | $\frac{1}{2}\left[\left(\frac{L}{mV}-1\right)\frac{1}{E_{\mathrm{g,m}}}+\frac{L}{mV}+1\right]$ |
| d) $\dfrac{x_n-\frac{y_n}{m}}{x_n-x_{n-1}}$ | $\left(\frac{L}{mV}-1\right)\left(\frac{1}{E_{\mathrm{g1}}}-1\right)$ | 0 | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{g3}}}-1\right)$ | $1-\frac{1}{E_{\mathrm{g4}}}$ | $\frac{1}{2}\left(\frac{L}{mV}-1\right)\left(\frac{1}{E_{\mathrm{g,m}}}-1\right)$ |
| *Cross flow* | | | | | |
| a) $\dfrac{x_{n-1}-\frac{y_{n-1}}{m}}{x_n-x_{n-1}}$ | $\left(\frac{L}{mV}-\frac{1}{2}\right)\frac{1}{E_{\mathrm{k1}}}$ | $\frac{L}{mV}+\frac{1}{2E_{\mathrm{k2}}}-1$ | $\left(\frac{L}{mV}+\frac{1}{2}\right)\frac{1}{E_{\mathrm{k3}}}-1$ | $\frac{L}{mV}-\frac{1}{2E_{\mathrm{k4}}}$ | $\frac{1}{2}\left[\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k,m}}}+1\right)-1\right]$ |
| b) $\dfrac{x_{n-1}-\frac{y_n}{m}}{x_n-x_{n-1}}$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k1}}}-1\right)-\frac{1}{2E_{\mathrm{k1}}}$ | $\frac{1}{2E_{\mathrm{k2}}}-1$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k3}}}-1\right)+\frac{1}{2E_{\mathrm{k3}}}-1$ | $-\frac{1}{2E_{\mathrm{k4}}}$ | $\frac{1}{2}\left[\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k,m}}}-1\right)-1\right]$ |
| c) $\dfrac{x_n-\frac{y_{n-1}}{m}}{x_n-x_{n-1}}$ | $\left(\frac{L}{mV}-\frac{1}{2}\right)\frac{1}{E_{\mathrm{k1}}}+1$ | $\frac{L}{mV}+\frac{1}{2E_{\mathrm{k2}}}$ | $\left(\frac{L}{mV}+\frac{1}{2}\right)\frac{1}{E_{\mathrm{k3}}}$ | $\frac{L}{mV}-\frac{1}{2E_{\mathrm{k4}}}+1$ | $\frac{1}{2}\left[\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k,m}}}+1\right)+1\right]$ |
| d) $\dfrac{x_n-\frac{y_n}{m}}{x_n-x_{n-1}}$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k1}}}-1\right)-\frac{1}{2E_{\mathrm{k1}}}+1$ | $\frac{1}{2E_{\mathrm{k2}}}$ | $\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k3}}}-1\right)+\frac{1}{2E_{\mathrm{k3}}}$ | $1-\frac{1}{2E_{\mathrm{k4}}}$ | $\frac{1}{2}\left[\frac{L}{mV}\left(\frac{1}{E_{\mathrm{k,m}}}-1\right)+1\right]$ |

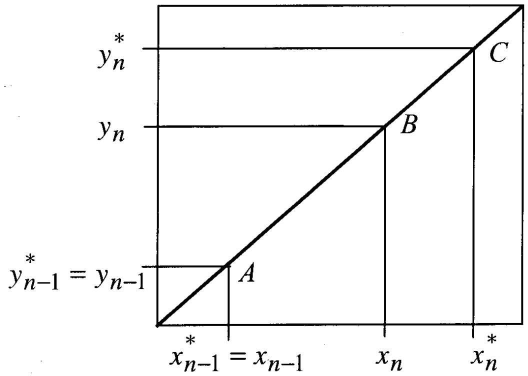


Fig. 1. Variation in the concentration in the second variant of mass exchange for backward flow.

TABLE 2. Limiting Values of $h$ and $h_1$ in the Complex Model

| Ratio | Form of organization of flows | | |
|---|---|---|---|
| | Forward flow | Backward flow | Cross flow |
| a) $\dfrac{x_{n-1} - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}}$ | $\dfrac{\dfrac{L}{mV} + 1 - E_f}{\left(\dfrac{L}{mV} + 1\right)(1 - E_f)}$ | $\dfrac{\dfrac{L}{mV} - E_g}{\left(\dfrac{L}{mV} + 1\right)(1 - E_g)}$ | $\dfrac{\dfrac{L}{mV} + \dfrac{1}{2} - E_k}{\left(\dfrac{L}{mV} + 1\right)(1 - E_k)}$ |
| b) $\dfrac{x_{n-1} - \dfrac{y_n}{m}}{x_n - x_{n-1}}$ | 1 | $\dfrac{\dfrac{L}{mV} - \left(\dfrac{L}{mV} + 1\right)E_g}{\left(\dfrac{L}{mV} + 1\right)(1 - E_g)}$ | $\dfrac{\dfrac{L}{mV} + \dfrac{1}{2} - \left(\dfrac{L}{mV} + 1\right)E_k}{\left(\dfrac{L}{mV} + 1\right)(1 - E_k)}$ |
| c) $\dfrac{x_n - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}}$ | $\dfrac{1}{1 - E_f}$ | $\dfrac{\dfrac{L}{mV}}{\left(\dfrac{L}{mV} + 1\right)(1 - E_g)}$ | $\dfrac{\dfrac{L}{mV} + \dfrac{1}{2}}{\left(\dfrac{L}{mV} + 1\right)(1 - E_k)}$ |
| d) $\dfrac{x_n - \dfrac{y_n}{m}}{x_n - x_{n-1}}$ | $\dfrac{\dfrac{L}{mV}(1 - E_f) + 1}{\left(\dfrac{L}{mV} + 1\right)(1 - E_f)}$ | $\dfrac{\dfrac{L}{mV}}{\dfrac{L}{mV} + 1}$ | $\dfrac{\dfrac{L}{mV}(1 - E_k) + \dfrac{1}{2}}{\left(\dfrac{L}{mV} + 1\right)(1 - E_k)}$ |

quently, the complex model in forward flow holds throughout the range of variation of the distances $h$ and $h_1$ from zero to values less than unity.

The data of Table 1 show that in the case of backward flow the ratios of the concentration difference in the second variant are independent of the efficiency and the efficiency cannot be determined. The equality of ratio b to minus unity and of ratio d to zero together with the conditions of equilibrium of the vapor and liquid phases on the ideal plate $y_n^* = mx_{n-1}^*$ show the identity of the ideal and real plates in this variant. Furthermore, the negative value of ratio b indicates that in the second variant we have $y_n/m > x_{n-1}$.

In the fourth variant, the concentration ratios b and d have negative values and ratios a and c can be the same for $L/(mV) < 1$ and $E_g[L/(mV) + 1] < 1$ respectively. The reason is that the coefficient of phase equilibrium tends to zero, $h$ and $h_1$ tend to unity, and the concentration (divided by $m$) of one component of the mixture in the vapor phase exceeds the corresponding content of this component in the liquid.

In the first and third variants of mass exchange and in the variant of separation of an ideal mixture ($h = h_1 = 0.5$), the negative ratios of the concentration differences are not observed in explicit form. However, as applied to the first variant, we must note the following. For this variant one observes the coincidence of the compositions of the vapor arriving at ideal and real plates and of the liquid flowing from them. On the $y$–$x$ plot (Fig. 1), the equilibrium line passes through points $A$ and $C$ with coordinates $y_{n-1}^*$, $x_{n-1}^*$, and $y_n^*$, $x_n^*$ respectively. The working straight line

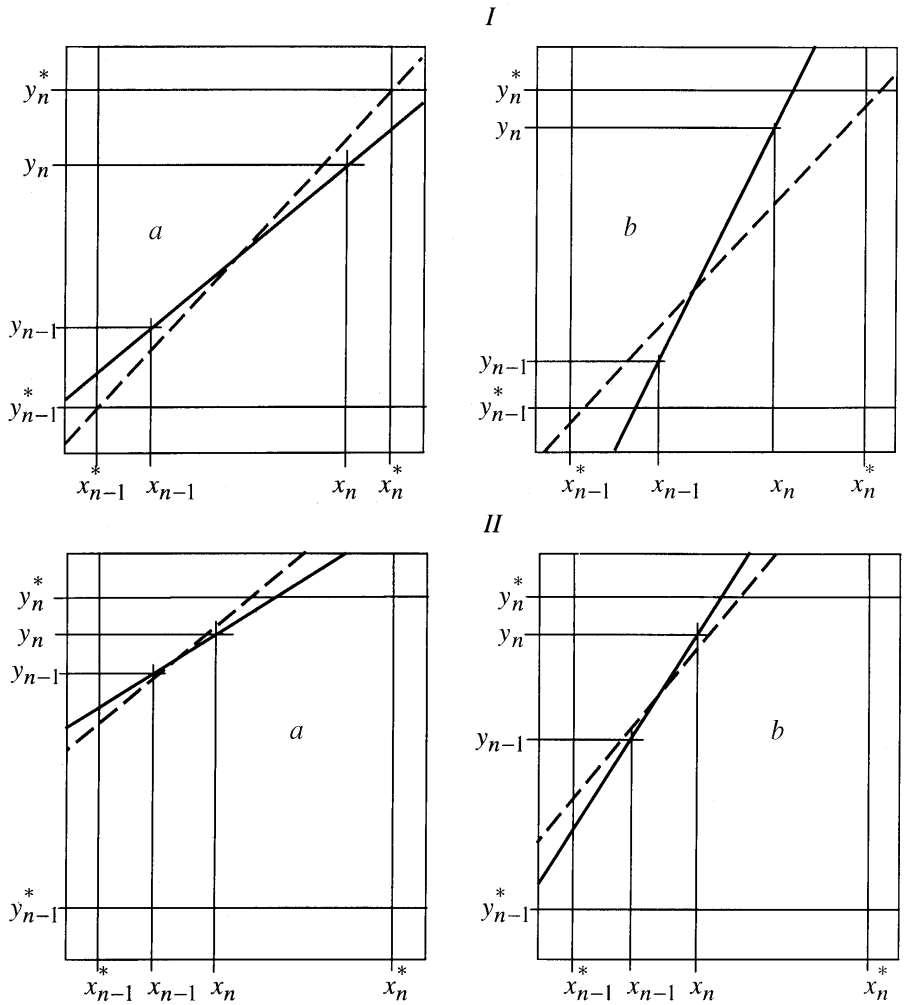


Fig. 2. Features of the variation in the concentration for backward flow (I) and cross flow (II) in the fourth variant of mass exchange on ideal (dashed lines) and real (solid lines) plates: a) $x_n > y_n/m$ and $x_{n-1} < y_{n-1}/m$; b) $x_n < y_n/m$ and $x_{n-1} > y_{n-1}/m$.

passes through points $A$ and $B$ with coordinates $y_{n-1}$, $x_{n-1}$, and $y_n$, $x_n$ respectively. As the efficiency increases to unity, point $B$, moving along the straight line $AB$, will coincide with point $C$, which belongs to the equilibrium line. If we allow for the rectilinearity of the equilibrium line on the portion of variation of the concentrations on the plate, we obtain the coincidence of the working and equilibrium lines, i.e., the same angle of inclination of these lines. As a consequence, the ratios of the concentration difference a and b are equal to zero and it is impossible to determine the efficiency in this variant (just as in the second variant). This circumstance together with certain limitations of the second variant in forward flow (ratio b) emphasizes the lack of logic in the conditions of interrelationship between the ideal and real plates inherent in the Murphree model and eliminates the first and second variants of mass exchange from the number of working models in the case of backward flow.

Thus, we should assume the third and fourth variants to be the limiting cases in backward flow. Separation of an ideal mixture can be considered to be the intermediate state of the complex model between these variants.

In cross flow (Table 1), ratio b has a negative value in the fourth variant and the analogous value is possible for ratio d, when $E_k > 0.5$, which points to the excess of $y_n/m$ over $x_{n-1}$ and, when $E_k < 0.5$, also over $x_n$. In the second variant, the value of ratio b can be negative when $E_k > 0.5$. Consequently, the distances $h$ and $h_1$ must take on lower values and cannot be equal to unity. Other ratios in the indicated variants and in other variants are assumed to be positive.

TABLE 3. Limiting Values of the Technological Parameters

| Ratio | Parameter | Form of organization of flows | | |
|---|---|---|---|---|
| | | Forward flow | Backward flow | Cross flow |
| a) $\dfrac{x_{n-1} - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}}$ | $\dfrac{L}{mV}$ | $-\dfrac{m(1 - E_f)}{m + E_f}$ | $\dfrac{mE_g + 1}{m + E_g}$ | $\dfrac{mE_k - \dfrac{m-1}{2}}{m + E_k}$ |
| | $E$ | $\dfrac{m + \dfrac{L}{V}}{m - \dfrac{L}{mV}}$ | $\dfrac{\dfrac{L}{V} - 1}{m - \dfrac{L}{mV}}$ | $\dfrac{\dfrac{L}{V} + \dfrac{m-1}{2}}{m - \dfrac{L}{mV}}$ |
| b) $\dfrac{x_{n-1} - \dfrac{y_n}{m}}{x_n - x_{n-1}}$ | $\dfrac{L}{mV}$ | –1 | $\dfrac{mE_g + 1}{m(1 - E_g)}$ | $\dfrac{mE_k - \dfrac{m-1}{2}}{m(1 - E_k)}$ |
| | $E$ | 1 | $\dfrac{\dfrac{L}{V} - 1}{\dfrac{L}{V} + m}$ | $\dfrac{\dfrac{L}{V} + \dfrac{m-1}{2}}{\dfrac{L}{V} + m}$ |
| c) $\dfrac{x_n - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}}$ | $\dfrac{L}{mV}$ | –1 | $\dfrac{1 - E_g}{m + E_g}$ | $-\dfrac{E_k + \dfrac{m-1}{2}}{m + E_k}$ |
| | $E$ | $-m$ | $-\dfrac{\dfrac{L}{V} - 1}{\dfrac{L}{mV} + 1}$ | $-\dfrac{\dfrac{L}{V} + \dfrac{m-1}{2}}{\dfrac{L}{mV} + 1}$ |
| d) $\dfrac{x_n - \dfrac{y_n}{m}}{x_n - x_{n-1}}$ | $\dfrac{L}{mV}$ | $-\dfrac{m + E_f}{m(1 - E_f)}$ | $\dfrac{1}{m}$ | $-\dfrac{E_k + \dfrac{m-1}{2}}{m(1 - E_k)}$ |
| | $E$ | $\dfrac{\dfrac{L}{V} + m}{\dfrac{L}{V} - 1}$ | 1 | $\dfrac{\dfrac{L}{V} + \dfrac{m-1}{2}}{\dfrac{L}{V} - 1}$ |

In backward and cross flows, as has been noted above, the ratios of the concentration differences a–d can have negative values. In this connection, it is expedient to find the limiting values of the distances $h$ and $h_1$ for the indicated forms of organization of the flows, including forward flow, by equating all the ratios in question to zero. The sought values derived from the right-hand sides of formulas (7)–(18) are given in Table 2.

In forward flow, the limiting values of $h$ and $h_1$ are equal to unity (ratio b) or exceed it, which is confirmed by the absence of negative values (Table 2). Consequently, these distances can take on any real values (except the limiting unity) in the case of forward flow.

In backward and cross motions of the vapor and the liquid, the limiting values of $h$ and $h_1$ can be less than unity, which points to the possibility of obtaining negative ratios of the concentration differences in the case where these distances exceed the values indicated in Table 2. In backward flow and cross flow, there can be the situations given in Fig. 2 when one ratio is positive and the other is negative. The same situation is also observed for other ratios of the concentration differences which are not given in Fig. 2. This circumstance should be taken into account in selecting the determining values of the distances $h$ and $h_1$, which, probably, must be lower that the minimum value (ratio b) for the coefficient of phase equilibrium exceeding unity and higher than the maximum value (ratio c) for $m < 1$. In the interval between the indicated values of the distances, preference should, possibly, be given to the ratio or its numerator employed in running calculations.

We emphasize that the distances $h$ and $h_1$ themselves are calculated from formula (6) and the analysis made is necessary for determining the limits of the complex model. In substituting the values of these distances from (6) into formulas (7)–(18), we obtain the limiting relations between the efficiency and $L/(mV)$ (Table 3) for the forms of organization of the flows in question. In particular, in forward flow, there are no limitations imposed on the values of $L/(mV)$ since this ratio is obviously higher than the values indicated in Table 3. The efficiency in forward flow can also vary within its natural limits without any limitations. In backward flow and cross flow, it is important to track the relation of the efficiency and $L/(mV)$. The excess of these values over the tabulated values points to the possibility of obtaining negative ratios of the concentration difference. On the whole, using the data of Table 3 one can evaluate the efficiency of mass exchange and the ratio of the flows.

Thus, the results of the analysis made enable one to estimate the values of the most important technological parameters and to determine the validity range for the complex model.

## NOTATION

$E$, efficiency of the plate; $h$ and $h_1$, dimensionless distances from the site of injection of the vapor and the liquid, respectively, to the surface of equality of the concentrations of the phases on ideal and real plates; $L$, molar flow of the liquid; $m$, coefficient of phase equilibrium; $V$, molar flow of the vapor; $x$ and $y$, concentrations of the highly volatile component in the liquid and the vapor respectively. Subscripts: g, backward flow; k, cross flow; m, values of the parameters for $h = h_1 = 0.5$; $n$, No. of the plate in question; $n - 1$, No. of the preceding plate in the direction of motion of the vapor; f, forward flow; $^*$, ideal conditions.

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